3.3 Videos Guide

3.3a

Exercise:

- Analysis of the graph of $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

- Increasing/decreasing test
  - (a) If $f'(x) > 0$ on an interval, then f is increasing on that interval.
    (b) If $f'(x) < 0$ on an interval, then f is decreasing on that interval.
- The First Derivative Test
  - Suppose that c is a critical number of a continuous function f.
    (a) If f' changes from positive to negative at c, then f has a local maximum at c.
    (b) If f' changes from negative to positive at c, then f has a local minimum at c.
    (c) If f' is positive to the left and right of c, or negative to the left and right of c, then f has no local maximum or minimum at c.

3.3b

Definition: (concave upward/concave downward)

- If the graph of f lies above all of its tangents on an interval I, then f is called concave upward on I. If the graph of f lies below all of its tangents on I, then f is called concave downward on I.

- Concavity Test
  - (a) If $f''(x) > 0$ on an interval I, then the graph of f is concave upward on I.
    (b) If $f''(x) < 0$ on an interval I, then the graph of f is concave downward on I.

Definition: (inflection point)

- A point P on a curve $y = f(x)$ is called an inflection point if f is continuous there and the curve changes from concave upward to concave downward or from concave downward to concave upward at P.

- The Second Derivative Test
  - Suppose f'' is continuous near c.
    (a) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.
    (b) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.

3.3c

Exercise:

- For the function $f(x) = 5x^{2/3} - 2x^{5/3}$, find the following.
  (a) Intervals on which f is increasing or decreasing.
  (b) Local maximum and minimum values of f.
  (c) Intervals of concavity and the inflection points.

  (d) Then use the information from parts (a)-(c) to sketch the graph.